Warsaw University of Technology

Faculty of Power and Aeronautical Engineering

WARSAW UNIVERSITY OF TECHNOLOGY

Institute of Aeronautics and Applied Mechanics

Finite element method 2 (FEM2)

Error estimation in a finite element

A-POSTERIORI ERROR APPROXIMATION

VECTOR OF THE AVERAGE STRESS AT NODE i :

$$\{\sigma\}_i^{ave} = \frac{\sum_{e=1}^{k} \{\sigma\}_i^e}{k}$$

where :

k – number of elements adjacent to node i

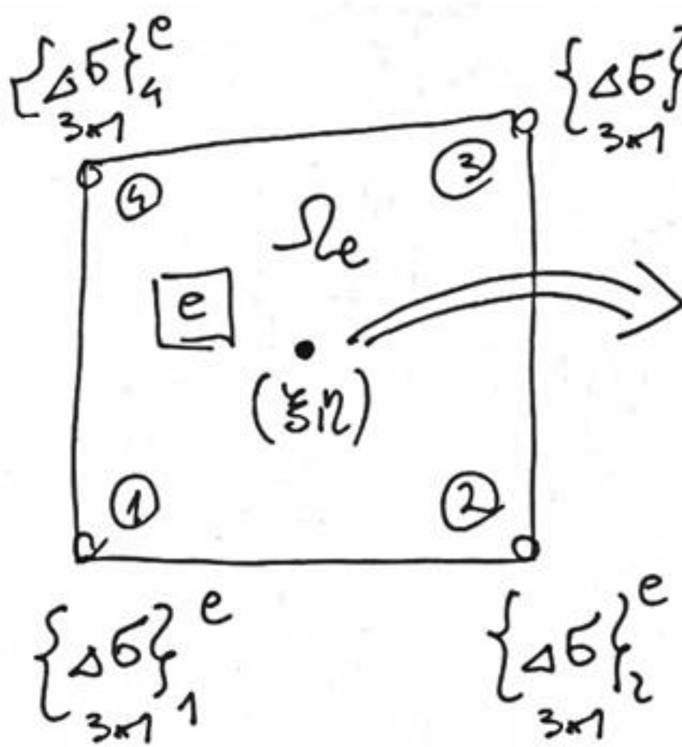
e – element number

$$\underset{6\times 1}{\{\sigma\}_i^e} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}_i^e$$

THE STRESS ERROR IN ELEMENT e AT NODE i :

$$\underset{6\times 1}{\{\Delta\sigma\}_i^e} = \underset{6\times 1}{\{\sigma\}_i^e} - \{\sigma\}_i^{ave}$$

NODAL APPROXIMATION OF THE STRESS ERROR (QUAD 4 -NODE, PLANE STRESS)



$$\underset{3\times 1}{\{\Delta\sigma(\xi,\eta)\}^e} = \sum_{i=1}^{4} \{\Delta\sigma\}_i^e \cdot N_i(\xi,\eta)$$

The maximum absolute value of any stress component :

$$\max\left(\{\Delta\sigma\}^{e}_{\substack{i \\ 6\times 1}} \right) \qquad (\text{SDSG} - \text{IN ANSYS})$$

Strain energy in a finite element :

$$U_e = \frac{1}{2} \int_{\Omega_e} \lfloor \sigma \rfloor^{e}_{1\times 6} \cdot \{\varepsilon\}^{e}_{6\times 1} \, d\Omega_e = \frac{1}{2} \int_{\Omega_e} \lfloor \sigma \rfloor^{e}_{1\times 6} \underbrace{[D]^{-1}_{6\times 6}}_{\text{Inverse constitutive matrix}} \{\sigma\}^{e}_{6\times 1} \, d\Omega_e$$

strain energy density in a finite element

$$u_e = \frac{U_e}{\int_{\Omega_e} d\Omega_e} \qquad (\text{SENDELAS} - \text{IN ANSYS})$$

Total strain energy for the entire model:

$$U = \sum_{e=1}^{NOE} U_e$$

strain energy error in a finite element:

$$\Delta U_e = \frac{1}{2} \int_{\Omega_e} \underset{1\times 6}{\lfloor \Delta\sigma \rfloor^e} \underset{6\times 6}{[D]^{-1}} \cdot \underset{6\times 1}{\{\Delta\sigma\}^e} \, d\Omega_e \qquad \text{(SERR-IN ANSYS)}$$

Total strain energy error in the entire model:

$$\Delta U = \sum_{e=1}^{NOE} \Delta U_e$$

The normalized energy error (for the entire model) is calculated as:

$$S = 100 \cdot \sqrt{\frac{\Delta U}{\Delta U + U}} \qquad \text{(SEPC IN ANSYS)}$$

The procedure of mesh refining:

$\Delta U_e = \text{const}$ for all FEs – „error equilibration"

$S < S_0$ for the entire model

S_0 – allowable value of S.

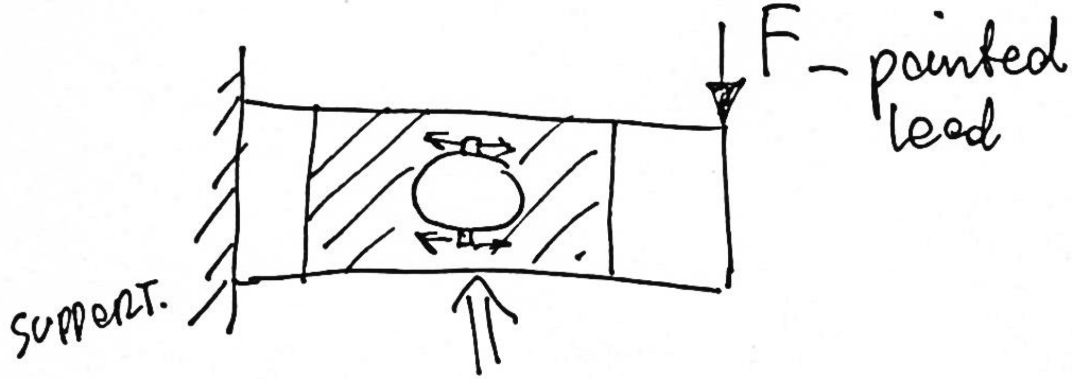
1°) Adaptive meshing – to correct mesh until:

$$\frac{\Delta U_e - \frac{\Delta U}{NOE}}{\frac{\Delta U}{NOE}} < \Delta U_0$$

allowable value.

and $S < S_0$

2°) Selective adaptive meshing



TYPES OF REFINEMENT:

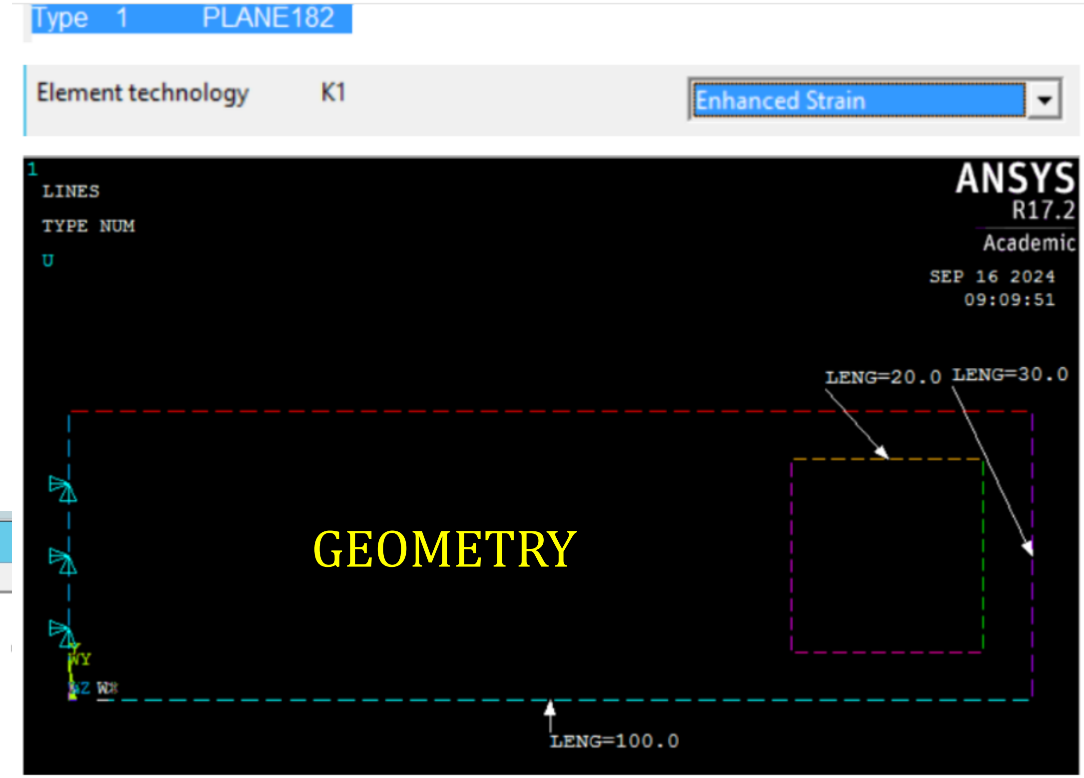
h – size decrease

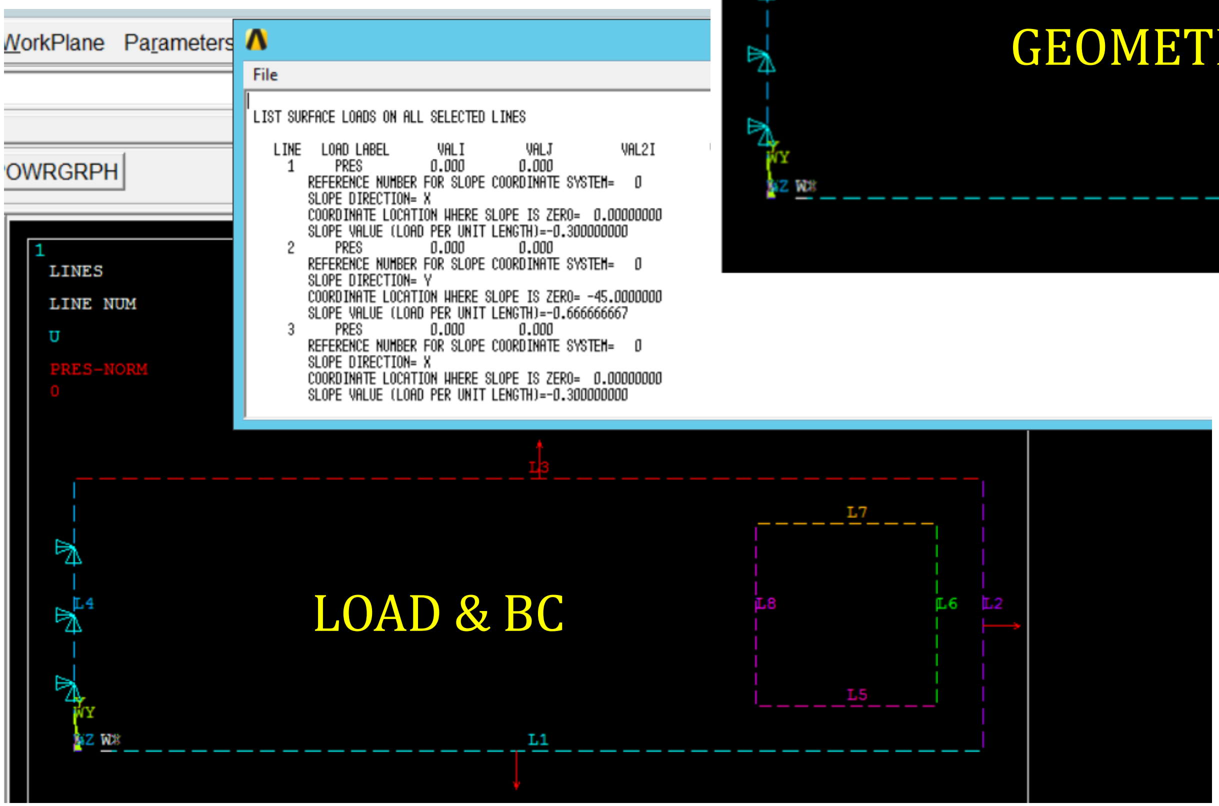
p – polynomal order in shape functions

r – nodes rearrangement

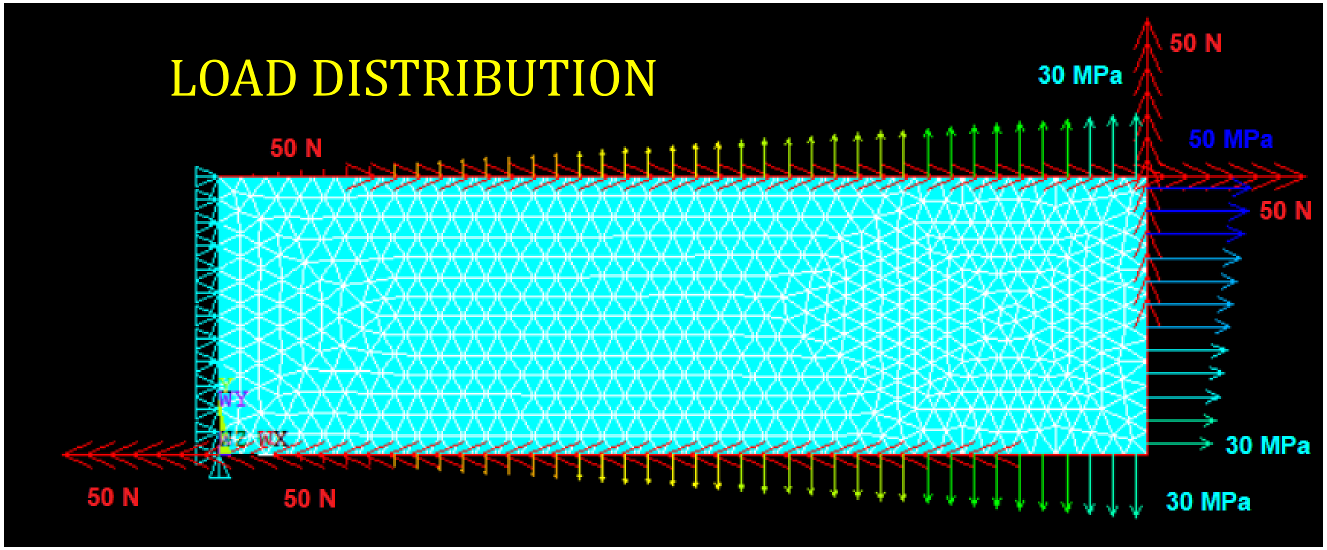
h_p – combination of „h" and „p"

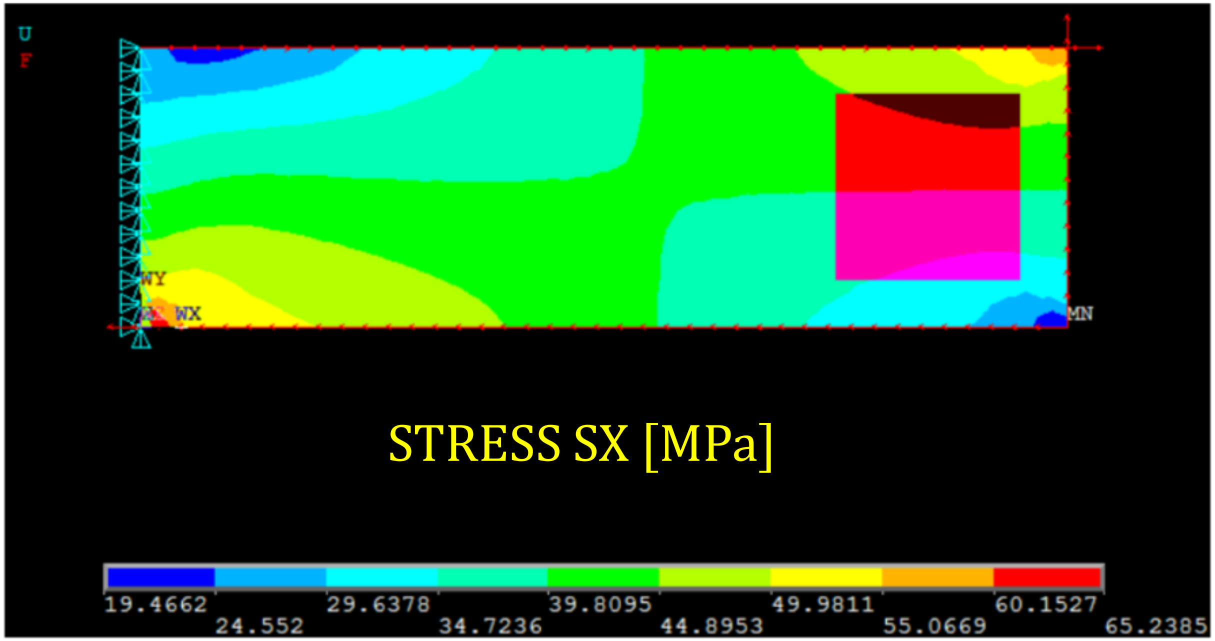
Example. Plate



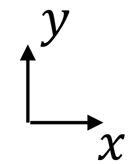


Example. Plate

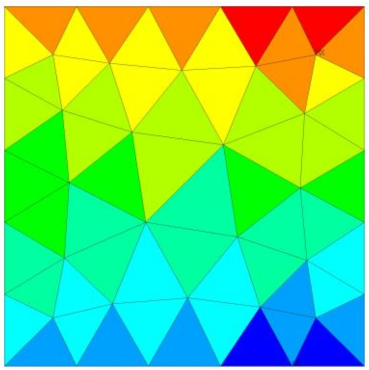




Normal stress in x direction (SX)

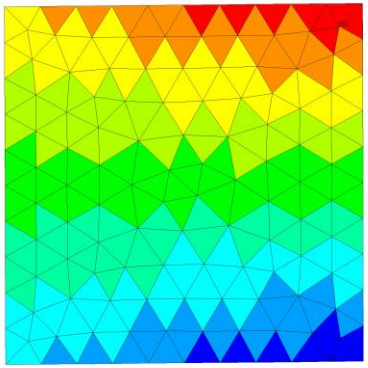


CST, 4mm
(free mesh)



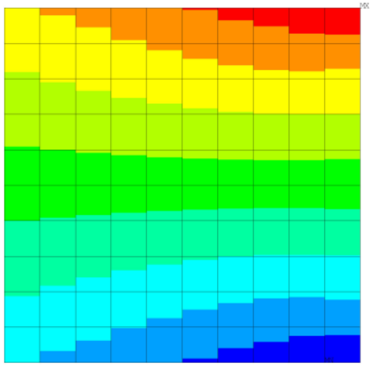
SMN =29.9391
SMX =50.0735

CST, 2mm
(free mesh)

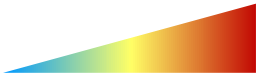


SMN =31.3547
SMX =48.657

QUAD-4node, 2mm
(mapped mesh)

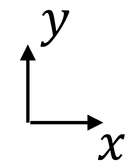


SMN =31.2497
SMX =48.819

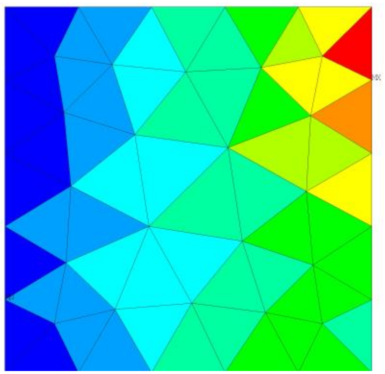


Min Max

Normal stress in y direction (SY)

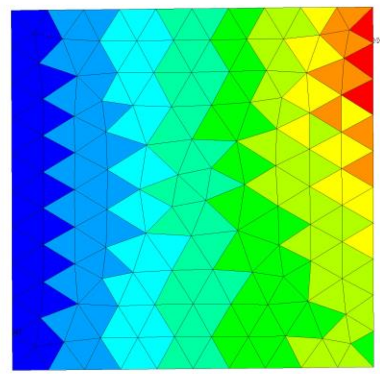


CST, 4mm



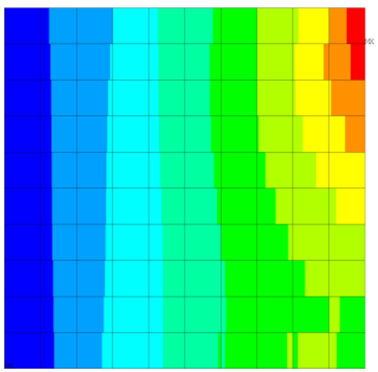
SMN =22.4161
SMX =31.2216

CST, 2mm



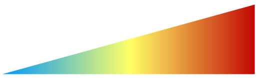
SMN =22.4077
SMX =30.4297

QUAD-4node, 2mm

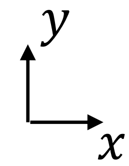


SMN =22.3724
SMX =30.5656

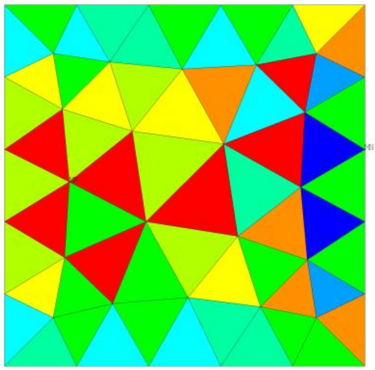
(MPa)



Shear stress (SXY)

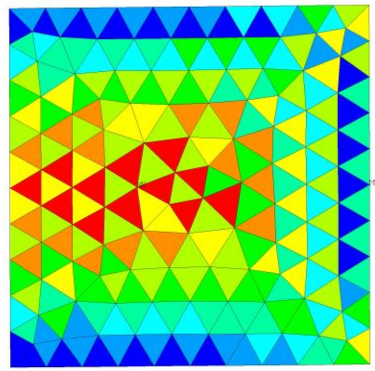


CST, 4mm



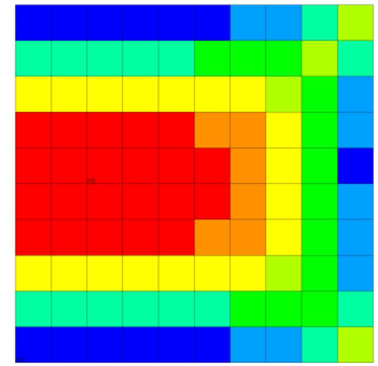
SMN =20.9712
SMX =22.9216

CST, 2mm



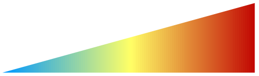
SMN =21.4276
SMX =22.7416

QUAD-4node, 2mm

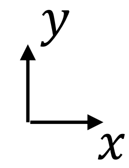


SMN =21.553
SMX =22.4885

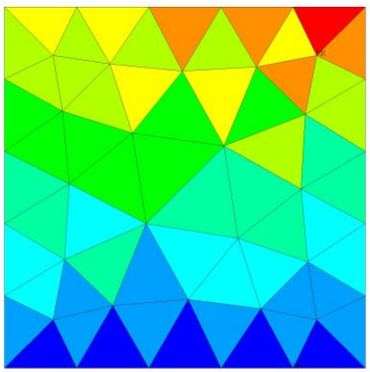
(MPa)



Von Mises stress (SEQV)

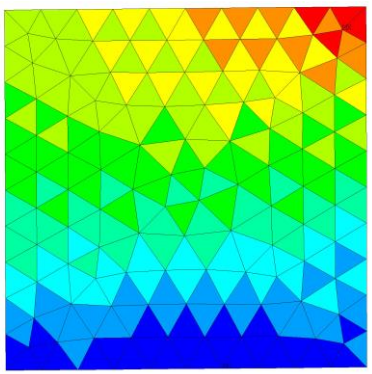


CST, 4mm



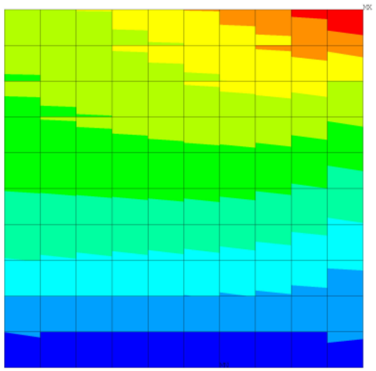
SMN =47.474
SMX =58.2418

CST, 2mm



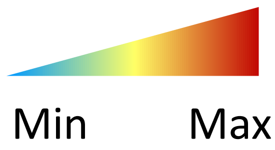
SMN =47.9007
SMX =57.2223

QUAD-4node, 2mm



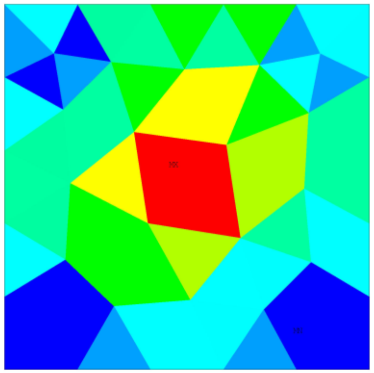
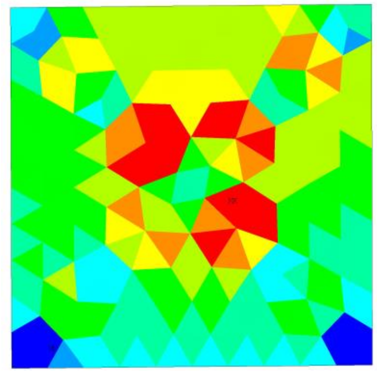
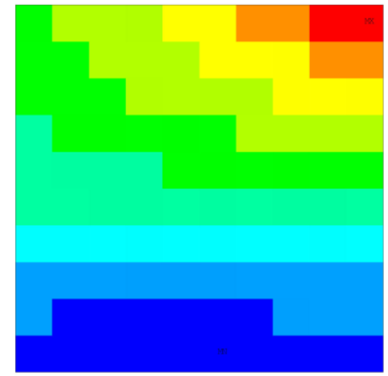
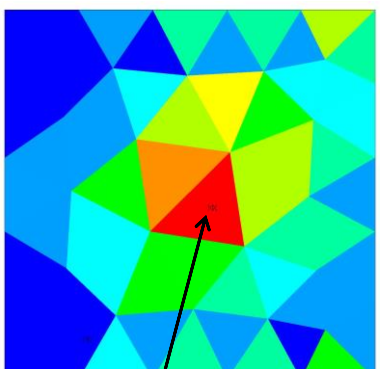
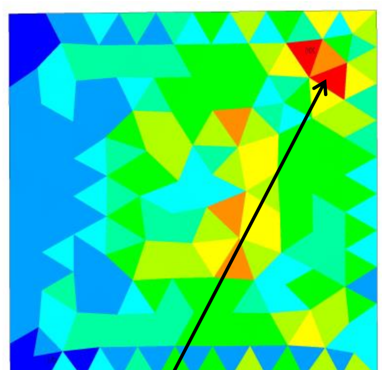
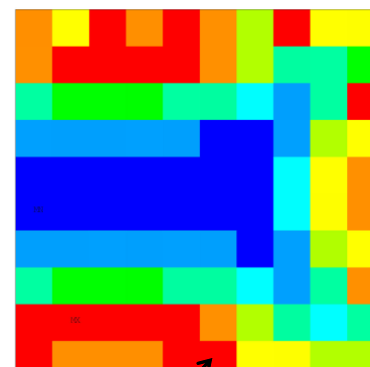
SMN =47.9151
SMX =57.431

(MPa)



Elastic strain energy (SENE)

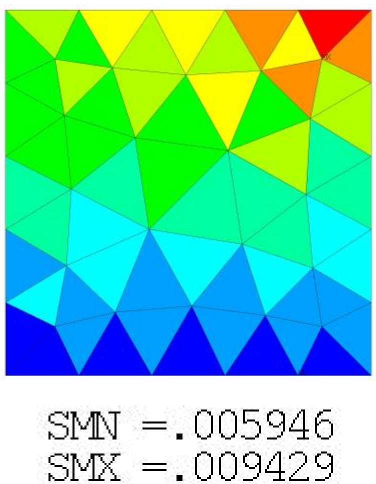
$$U_e = \frac{1}{2}\int_{\Omega_e} \underset{1\times 6}{\lfloor \varepsilon \rfloor} \underset{6\times 1}{\{\sigma\}}\, d\Omega_e \quad \text{(Nmm)}$$

	CST, 4mm	CST, 2mm	QUAD-4node, 2mm
(SENE)	 SMN =.029352 SMX =.093964	 SMN =.007026 SMX =.019023	 SMN =.024947 SMX =.035708
Error (SERR)	 SMN =.944E-05 SMX =.155E-03 (SEPC= 4.06%)	 SMN =.434E-06 SMX =.624E-05 (SEPC= 1.92%)	 SMN =.309E-07 SMX =.945E-06 (SEPC= 0.51%)

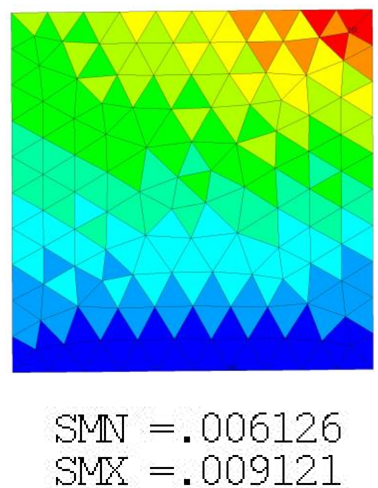
Elastic strain energy density (SENDELAS)

$$u_e = \frac{U_e}{\int_{\Omega_e} d\Omega_e}$$

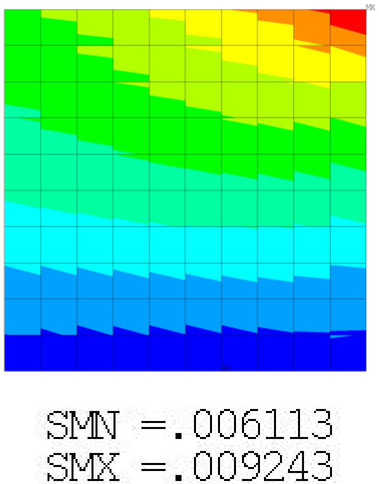
CST, 4mm



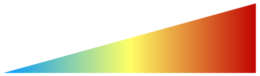
CST, 2mm



QUAD-4node, 2mm

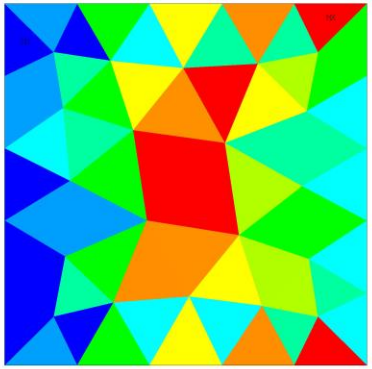
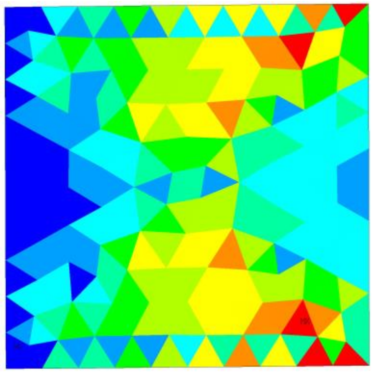
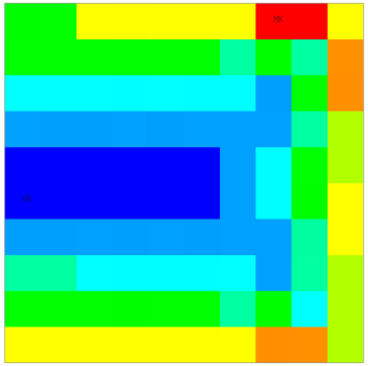


(Nmm/mm³)



Min Max

Maximum absolute value of error of any stress component (SDSG)

CST, 4mm	CST, 2mm	QUAD-4node, 2mm
		
SMN =1.07305 SMX =3.37362	SMN =.490022 SMX =1.61018	SMN =.045321 SMX =.338378

$$\max\left(\{\Delta\sigma\}_{i}^{e}\right)_{6\times 1}$$

(MPa)